Paper No.
69-LubS-8

Influence of Flexibly Mounted Rolling Element Bearings on Rotor Response Part 1—Linear Analysis

E. J. GUNTER
Associate Professor,
Department of Mechanical Engineering,
University of Virginia, Charlottesville,
Va. Mem. ASME

The paper evaluates the influence of damped, linear flexibly mounted rolling-element bearings on dynamic rotor unbalance response. The system analyzed is treated as a general four degree of freedom unbalanced rotor mounted on damped flexible supports and includes rotor gyroscopic effects. The rotor equations of motion are solved for synchronous precession over a wide range of speeds for various support conditions. Rotor performance curves on bearing amplitude, forces transmitted, phase angles as a function of speed for various values of support damping are computer plotted to illustrate rotor and bearing performance over a wide range of speed and operating parameters. Results indicate that forces transmitted to the bearings by the rotor synchronous unbalance response can be dramatically reduced by proper design of the bearing support characteristics.

Introduction

THE present design philosophy in commercial gas turbine applications is to extend the unit operating life and also the operating hours between scheduled inspections and down-time. The gas turbine life is influenced by such factors as the stress-rupture life of the turbine blades and the life of the bearings. The rolling element bearing life is a function of speed and of the applied axial and radial forces. These various bearing forces may be due to bearing preload, turbine and compressor aerodynamic forces, gyroscopic loads caused by aircraft maneuvers, or dynamic loading caused by rotor unbalance. It is the purpose of this paper to deal predominantly with the transmitted bearing forces caused by rotor unbalance. The rotor unbalance may excite one or more critical speeds of the rotor itself, and may also induce resonances in the supporting structure.

The problem of minimization of the rotor unbalance response becomes particularly acute in the design of high speed lightweight rolling element supported gas turbines which must operate smoothly over an extended speed range. One obvious method to limit the rotor amplitude is to design the rotor and support structure relatively stiff so as to place the majority of the critical speeds above the operating range. This method was found desirable by Alford [1],[1] in the case of the turbine instability encountered with several different designs of aircraft engines which all had relatively large rotor flexibility and low critical speeds. In all cases the stability was improved by increasing the rigidity of the turbine and support system.

Several manufacturers [2], [3], [4], [5] have reported that a smooth engine may be obtained by mounting the rolling element bearings in a flexible spring mount to control the bearing forces transmitted. Van Nimwegen reports on good success achieved with an undamped flexible mount which allows a maximum amount of travel from 0.0025 to 0.005 in. The flexible support system thus functions as a bilinear spring, controlling the rotor amplitudes at resonance [5]. The stiffness of the flexible ring can be carefully designed to locate the critical speeds at suitable values so as not to produce large resonant vibrations in the rotor operating speed range.

Hamburg and Parkinson [6] report on effective use of a damped flexibly mounted ball bearing in controlling the turbine vibrational amplitude. In this design the outer race of the ball bearing is mounted in a flanged flexible cylinder which is bolted to the rotor casing. The cylindrical support is separated from the stationary housing by a thin film of oil to form a squeeze film bearing. E. W. Snow, in his comments to Alford's paper [1], mentions the successful use of an oil film between the bearing and housing on Rolls Royce engines. Satisfactory performance was obtained with respect to both synchronous precession caused by unbalance as well as self-excited nonsynchronous shaft whirling.

Hamburg reports that in the design of the Continental Model 217-S turboshaft engine a relatively stiff shaft was used and the resonant frequencies of the system controlled by regulating the spring rates of the bearing support structure. The normal

[1] Numbers in brackets designate References at end of paper.

Contributed by the Lubrication Division for presentation at the Lubrication Symposium, San Francisco, June 17–19, 1969, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, February 27, 1969. Paper No. 69-LubS-8.

Nomenclature

A = coefficient matrix
C = damping coefficient, lb-sec/in.
e_u = radial displacement of rotor mass center from axis of rotation, in.
$F_{Bx,y}$ = horizontal and vertical components of bearing force, lb
$F_{Sx,y}$ = horizontal and vertical components of support force, lb
Fu = unbalance force, lb
H_i = axial distance from the first bearing to the i'th unbalance station, in.
I_p = polar moment of inertia, lb-in.-sec^2
I_T = transverse moment of inertia about the rotor mass center, lb-in.-sec^2
K = stiffness coefficient, lb/in.
L = span between bearings, in.
L_1 = distance from the first bearing to the rotor mass center
L_2 = distance from the rotor mass center to the second bearing, in.
M = rotor mass, $\frac{\text{lb}}{\text{in.}}$ – sec^2
$M_{x,y}$ = unbalance moment about x, y axis, in.-lb
$m_{1,2}$ = bearing housing masses, $\frac{\text{lb}}{\text{in.}}$ – sec^2
$R_{1,2}$ = radial displacement of the unbalance masses, in.
t = time

(Continued on next page)

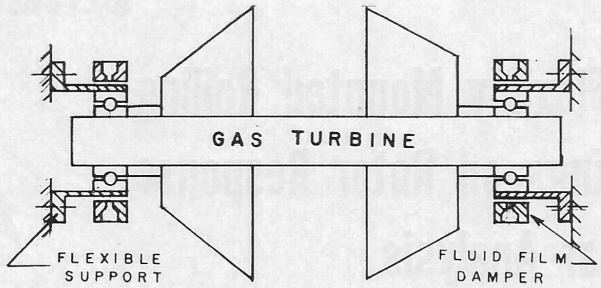

Fig. 1 Schematic diagram of the general rigid body rotor

operating range was free from critical speeds with the first and second modes below the idle speed. The third or flexible free-free critical speed (corresponding to the case of 0 bearing stiffness) was placed well above the operating speed range. Under these design conditions, the rotor first two critical speeds are essentially rigid body modes as shown by Suter [7] and Linn and Prohl [8]. Cooper [9], a research engineer with Rolls Royce, reported in 1963 on the experimental use of the hydrodynamic squeeze film bearing incorporated in a rolling element test rotor. He noted that the oil film provided good attenuation of the rotor unbalance response, but when a larger squeeze film bearing of identical clearance was used, it appeared that the higher film forces developed would not permit inversion through to the critical speed. Rolls Royce has extended this work and is presently using a squeeze film mounting on the Vickers Super VC10 Conway engines [10].

Although a number of companies have employed damped and undamped flexibly mounted bearings on gas turbines, little analysis has been presented in the literature as to the desirable range of stiffness and damping characteristics that the bearing supports should possess to minimize the amplitudes and forces transmitted over a given speed range. It is the intent of this paper to present quantitative information on desirable values of stiffness and damping of bearing mounts. In this particular paper only linear support stiffness and damping characteristics will be taken into consideration. In a squeeze film bearing as used by several companies, the hydrodynamic forces generated are highly nonlinear (as shown by Wood [11]) and therefore the analysis presented here is valid only for small amplitudes of motion for this type of bearing.

Rotor Equations of Motion

It is assumed that the gas turbine or rotor is designed to be relatively stiff and behave as a rigid body throughout the operating speed range. This assumption is valid only if the rotor free-free flexible critical speed based on zero bearing stiffness is considerably above the design speed.

Six degrees of freedom are required to represent the rigid rotor. The rotor governing equations of motion may be expressed in terms of the displacements and rotations of the rotor mass center or in terms of the x, y, z displacements of the two bearings (see Fig. 1).

The general rotor equations of motion including rotor acceleration are highly nonlinear and represent a difficult system of equations to solve analytically. These equations may be considerably simplified if one assumes small bearing displacements, constant rotor angular velocity, and neglects the rotor axial motion. These assumptions effectively reduce the number of rotor equations of motion from 6 to 4. Thus it is assumed that:

$$\dot{\alpha}_3 = \omega = \text{const} \qquad \alpha_3 = \omega t$$

$$\sin \alpha_1 \approx \alpha_1 = \frac{x_2 - x_1}{L} \ll 1 \text{ and}$$

$$\sin \alpha_2 \approx \alpha_2 = \frac{y_2 - y_1}{L} \ll 1$$

If it is assumed that the rotor unbalance is small in comparison to the rotor weight and also that the rolling element bearings are self aligning (so that no bearing moments are exerted on the rotor) then the rotor equations of motion are given as follows:

$$x_m: \quad M\ddot{x}_m = F_{Bx1} + F_{Bx2} + \delta M_1\omega^2 R_1 \cos \omega t + \delta M_2 \omega^2 R_2 \cos(\omega t + \phi) \qquad (1)$$

$$y_m: \quad M\ddot{y}_m = F_{By1} + F_{By2} + \delta M_1\omega^2 R_1 \sin \omega t + \delta M_2 \omega^2 R_2 \sin(\omega t + \phi) \qquad (2)$$

$$\alpha_1: \quad I_T\ddot{\alpha}_1 + \omega\dot{\alpha}_2 I_p = L_2F_{Bx2} - L_1F_{Bx1} + \delta M_1\rho_1R_1\omega^2 \cos \omega t + \delta M_2\rho_2R_2\omega^2 \cos(\omega t + \phi) \qquad (3)$$

$$\alpha_2: \quad I_T\ddot{\alpha}_2 - \omega\dot{\alpha}_1 I_p = L_2F_{By2} - L_1F_{By1} + \delta M_1\rho_1R_1\omega^2 \sin \omega t + \delta M_2\rho_2R_2\omega^2 \sin(\omega t + \phi) \qquad (4)$$

The four bearing housing equations of motion are given as follows:

$$x_{Bi}: \quad m_i\ddot{x}_{Bi} = F_{Bxi} - F_{sxi}; \quad i = 1, 2 \qquad (5)$$

$$y_{Bi}: \quad m_i\ddot{y}_{Bi} = F_{Byi} - F_{syi}; \quad i = 1, 2 \qquad (6)$$

If the mass m_i of the bearing housing is neglected then the rolling element bearing reaction may be equated to the support reaction

$$\left.\begin{aligned} F_{Bxi} &= F_{sxi} \\ F_{Byi} &= F_{syi} \end{aligned}\right\}; \quad i = 1, 2 \qquad (7)$$

If angular misalignment is not considered for the damped flexible support, then the most general support system such as a linearized squeeze film damper will require 4 stiffness and 4

Nomenclature

- TRD = dynamic transmissibility = F_B/Fu
- U = column vector of unbalance components
- x,y = shaft absolute horizontal and vertical displacement, in.
- x_M,y_M = absolute displacement of rotor mass center, in.
- x_b,y_b = bearing horizontal and vertical displacement, in.
- $x_{1,2}y_{1,2}$ = absolute shaft displacements at the first and second bearing locations
- z = shaft axial coordinate measured from the first bearing, in.
- α_1 = angular shaft displacement in the x-z plane, rad
- α_2 = angular shaft displacement in the y-z plane, rad
- α_3 = angular shaft rotation about z axis
- $\beta_{x,y}$ = phase angle between the unbalance force and the x or y displacements, deg
- β_α = phase angle between the unbalance moment and the angular shaft displacement, deg
- $\delta m_{1,2}$ = rotor unbalance masses, $\frac{\text{lb}}{\text{in.}}$ – sec^2
- ζ = dimensionless axial coordinate = z/L
- ρ = axial location at which unbalance forces act (measured from the rotor mass center), in.
- $\rho_{1,2}$ = axial location of the unbalance planes from the rotor mass center, in.
- ϕ = angular displacement of the radial unbalance planes, deg
- ω = shaft angular velocity, rad/sec

damping coefficients to represent the support.

$$F_{sxi} = -[C_{xx}\dot{x}_{Bi} + C_{xy}\dot{y}_{Bi} + K_{xx}x_{Bi} + K_{xy}Y_{Bi}]; \quad i = 1, 2 \quad (8)$$

$$F_{syi} = -[C_{yy}\dot{y}_{Bi} + C_{yx}\dot{x}_{Bi} + K_{yy}y_{Bi} + K_{yx}x_{Bi}]; \quad i = 1, 2 \quad (9)$$

If the rolling element stiffness is considerably greater then the support stiffness and damping values such that $k_x \gg k_{xx}$, ωC_{xx}, etc., then the relative bearing displacements may be neglected and the absolute rotor displacements x_i are equal to the absolute bearing displacements x_{Bi}.

Substituting:

$$\alpha_1 = \frac{x_2 - x_1}{L}; \qquad \alpha_2 = \frac{y_2 - y_1}{L}$$

$$x_m = \frac{L_1x_2 - L_2x_1}{L}; \quad y_m = \frac{L_1y_2 - L_2y_1}{L}$$

and equations (7)–(9) into equations (1)–(4), the linearized equations of motion of the four degree of freedom rigid body rotor in damped elastic supports are given by:

$$\frac{M}{L}(L_1\ddot{x}_2 + L_2\ddot{x}_1) + K_{1xx}x_1 + K_{2xx}x_2 + C_{1xx}\dot{x}_1 + C_{2xx}\dot{x}_2 + C_{1xy}\dot{y}_1 + C_{2xy}\dot{y}_2 + K_{1xy}y_1 + K_{2xy}y_2 = \delta M_1\omega^2R_1 \cos \omega t + \delta M_2\omega^2R_2 \cos (\omega t + \phi) \quad (10)$$

$$\frac{M}{L}(L_1\ddot{y}_2 + L_2\ddot{y}_1) + K_{1yy}y_1 + K_{2yy}y_2 + C_{1yy}\dot{y}_1 + C_{2yy}\dot{y}_2 + C_{1yx}\dot{x}_1 + C_{2yx}\dot{x}_2 + K_{1yx}x_1 + K_{2yx}y_2 = \delta M_1\omega^2R_1 \sin \omega t + \delta M_2\omega^2R_2 \sin (\omega t + \phi) \quad (11)$$

$$I_T\left(\frac{\ddot{x}_2 - \ddot{x}_1}{L}\right) + I_p\omega\left(\frac{\dot{y}_2 - \dot{y}_1}{L}\right) + C_{2xx}L_2\dot{x}_2 - C_{1xx}L_1\dot{x}_1C_{2xy}L_2\dot{y}_2 - C_{1xy}L_1\dot{y}_1 + K_{2xx}L_2x_2 - K_{1xx}L_1x_1 + K_{2xy}L_2y_2 - K_{1xy}L_1y_1 = \delta M_1\rho_1R_1\omega^2 \cos \omega t + \delta M_2\rho_2R_2\omega^2 \cos (\omega t + \phi) \quad (12)$$

$$I_T\left(\frac{\ddot{y}_2 - \ddot{y}_1}{L}\right) - I_p\omega\left(\frac{\dot{x}_2 - \dot{x}_1}{L}\right) + C_{2yy}L_2\dot{y}_2 - C_{1yy}L_1\dot{y}_1 + C_{2yx}L_2\dot{x}_2 - C_{1yx}L_1\dot{x}_1 + K_{2yy}L_2y_2 - K_{1yy}L_1y_1 + K_{2yx}L_2x_2 - K_{1yx}L_1x_1 = \delta M_1\rho_1R_1\omega^2 \sin \omega t + \delta M_2\rho_2R_2\omega^2 \sin (\omega t + \phi) \quad (13)$$

If the damped, flexible supports are formed by spring mounted, squeeze film hydrodynamic bearings then in general the cross coupling stiffness and damping terms will be present in the equations. Also note that if the rotor mass center is not symmetrically located in the bearing span, then different reactions may be expected from the two bearings.

Method of Solution

Assume a steady-state motion of synchronous rotor precession.

$$\left.\begin{aligned} x_i &= x_{ci} \cos \omega t + x_{si} \sin \omega t \\ y_i &= y_{ci} \cos \omega t + y_{si} \sin \omega t \end{aligned}\right\} \quad i = 1, 2 \quad (14)$$

Upon substituting the above displacement relationships into equations (10)–(13) the following eighth order matrix is obtained after equating the sin and cos coefficients of the four equations.

$$\begin{bmatrix} A_{11}\ A_{12} & \ldots & A_{18} \\ A_{21}\ A_{22} \ldots\ldots & & \\ A_{31} \ldots & & \\ \ldots & & \\ & & \\ & & \\ & & \\ A_{81} & & A_{88} \end{bmatrix} \begin{bmatrix} x_{c1} \\ x_{s1} \\ x_{c2} \\ x_{s2} \\ y_{c1} \\ y_{s1} \\ y_{c2} \\ y_{s2} \end{bmatrix} = \omega^2 \begin{bmatrix} \delta M_1R_1 + \delta M_2R_2 \cos \phi \\ -\delta M_2R_2 \sin \phi \\ \delta M_2R_2 \sin \phi \\ \delta M_1R_1 + \delta M_2R_2 \cos \phi \\ \delta M_1\rho_1R_1 + \delta M_2\rho_2R_2 \cos \phi \\ -\delta M_2\rho_2R_2 \sin \phi \\ \delta M_2\rho_2R_2 \sin \phi \\ \delta M_1\rho_1R_1 + \delta M_2R_2\rho_2 \cos \phi \end{bmatrix} \quad (15)$$

Or in matrix notation:

$$AX = U \quad (16)$$

The solution to the unknown column vector X is given by:

$$X = A^{-1}U \quad (17)$$

The steady state solution is therefore given by:

$$\left.\begin{aligned} x_i &= |x_i| \cos (\omega t - \psi_{xi}) \\ y_i &= |y_i| \sin (\omega t - \psi_{yi}) \end{aligned}\right\} \quad i = 1, 2 \quad (18)$$

where:

$$|x_i| = \sqrt{x_{ci}^2 + x_{si}^2}; \quad |y_i| = \sqrt{y_{ci}^2 + y_{si}^2}$$

$$\psi_{xi} = \tan^{-1}\left[\frac{x_{si}}{x_{ci}}\right]; \quad \psi_{yi} = \tan^{-1}\left[\frac{-y_{ci}}{x_{si}}\right]$$

The displacement X at any arbitrary point ζ measured along the shaft from the first bearing is given by:

$$x(\zeta) = |x| \cos (\omega t - \psi_x) \quad (19)$$

where:

$$|x| = \sqrt{(x_{c1}[1 - \zeta] + \zeta x_{c2})^2 + (x_{s1}[1 - \zeta] + \zeta x_{s2})^2}$$

$$\psi_x(\zeta) = \tan^{-1}\left(\frac{x_{s1}[1 - \zeta] + \zeta x_{s2}}{x_{c1}[1 - \zeta] + \zeta x_{c2}}\right)$$

Derivation of Phase Angles

The resultant exciting force due to the two planes of unbalance can be resolved into two components in the x and y directions as follows:

$$Fu_{x,y} = \delta M_1\omega^2R_1 \begin{Bmatrix} \cos \omega t \\ \sin \omega t \end{Bmatrix} + \delta M_2\omega^2R_2 \begin{Bmatrix} \cos (\omega t + \phi) \\ \sin (\omega t + \phi) \end{Bmatrix} \quad (20)$$

The unbalance force components can be expressed in terms of the total rotor mass M, the radial displacement e_u of the rotor mass center from the axis of rotation, and a phase angle ψ.

$$Fu_{x,y} = Me_u\omega^2 \begin{Bmatrix} \cos (\omega t + \psi) \\ \sin (\omega t + \psi) \end{Bmatrix} \quad (21)$$

where:

$$e_u = \frac{1}{M}\sqrt{(\delta M_1R_1)^2 + (\delta M_2R_2)^2 + 2\delta M_1\delta M_2R_1R_2 \cos \phi}$$

$$\psi = \tan^{-1}\left[\frac{\delta M_2R_2 \sin \phi}{\delta M_1R_1 + \delta M_2R_2 \cos \phi}\right]$$

Comparison of equations (18) and (21) shows that the rotating unbalance load leads the angular velocity vector by an angle of ψ while the rotor response lags the angular velocity by phase angles of ψ_x and ψ_y, respectively, for the x and y directions. Thus the phase angles between the rotating unbalance and the x and y rotor response are given by:

$$\beta_{x,y}(\zeta) = \psi + \psi_{x,y} \quad (22)$$

If β_x or β_y is positive, then the forcing function is leading the rotor response. Note that the phase angle will vary along the shaft and that the values observed at the first bearing will not

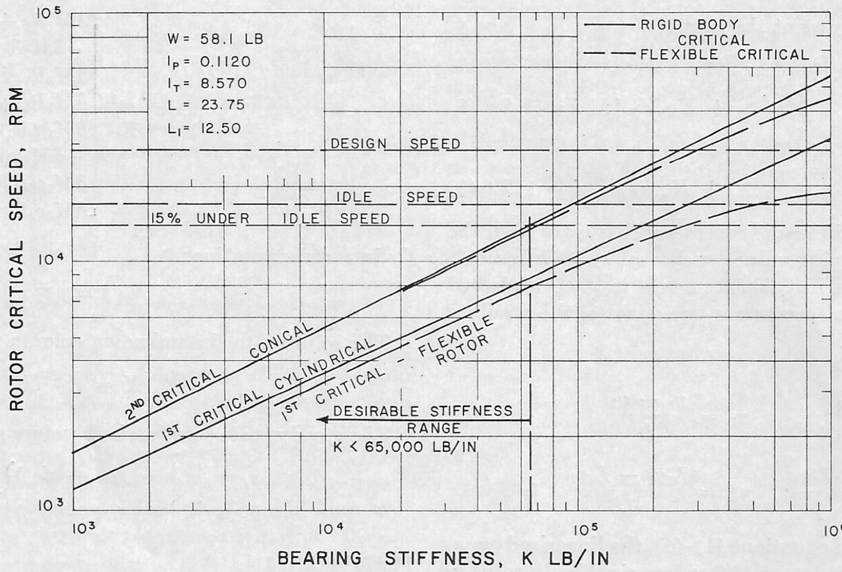

Fig. 2 Rotor critical speeds versus bearing stiffness

in general correspond to the values obtained at the second bearing location. If the bearing support characteristics are isotropic, then the phase angles in two orthogonal directions will be identical; that is $\beta_x = \beta_y = \beta$.

If the resultant rotor unbalance is not acting through a radial plane at the rotor mass center, then unbalance moments will be developed which will cause rotor conical motion.

The moments due to unbalance about the x and y axes are given by:

$$M_{x,y} = \delta M_1 R_1 \rho_1 \omega^2 \begin{Bmatrix} -\sin \omega t \\ \cos \omega t \end{Bmatrix} + \delta M_2 R_2 \rho_2 \omega^2 \begin{Bmatrix} -\sin(\omega t + \phi) \\ \cos(\omega t + \phi) \end{Bmatrix} \tag{23}$$

$$M_{x,y} = Me_u \rho \omega^2 \begin{Bmatrix} -\sin(\omega t + \psi_m) \\ \cos(\omega t + \psi_m) \end{Bmatrix} \tag{24}$$

where ρ represents the axial location of the radial plane (measured from the rotor mass center) at which the resultant of the unbalance forces act and is given by:

$$\rho = \frac{1}{Me_u} \sqrt{(\rho_1 R_1 \delta M_1)^2 + (\rho_2 R_2 M_2)^2 + 2\rho_1 \rho_2 R_1 R_2 \delta M_1 \delta M_2 \cos \phi}$$

$$\psi_m = \tan^{-1} \left[\frac{\rho_2 R_2 \delta M_2 \sin \phi}{\rho_1 R_1 \delta M_1 + \rho_2 R_2 \delta M_2 \cos \phi} \right]$$

The angular displacement of the rotor about the y axis is given by

$$\alpha_1 = \frac{x_2 - x_1}{L} = |\alpha_1| \cos(\omega t - \psi_{\alpha 1}) \tag{25}$$

where:

$$\alpha_1 = \frac{1}{L} \sqrt{(x_{c2} - x_{c1})^2 + (x_{s2} - x_{s1})^2}$$

$$\psi_{\alpha 1} = \tan^{-1} \left[\frac{x_{s2} - x_{s1}}{x_{c2} - x_{c1}} \right]$$

Hence the phase angle between the exciting unbalance moment and the conical response is given by:

$$\beta_{\alpha 1,2} = \psi_m + \psi_{\alpha 1,2} \tag{26}$$

A positive angular phase angle indicates that the exciting moment is leading the conical rotor response.

Computer Program "Rotor 4P"

A computer program was written to obtain the four degree of freedom steady-state rotor unbalance response as given by the preceding analysis. The equations of motion can be solved over a given speed range by specifying the initial speed, the speed increment, and the final speed. A total of eight linearized support stiffness and damping coefficients may be specified at each bearing location [12]. These coefficients may be either constant or speed dependent.

In addition to computing the rotor amplitudes of motion and the bearing forces transmitted, the phase angles of the rotor displacements and rotations relative to the excitation forces and moments caused by unbalance are also computed. The program also has the provision of computing the amplitude and phase angle at any arbitrary location along the shaft.

In a typical computer run, the rotor behavior may be calculated over a speed range of 0 to 30,000 rpm with a speed increment of 50 rpm. This will produce 600 values per variable to cover the specified speed range. For example, if two amplitudes are to be compared for three values of damping then there are 3600 points to plot for this particular graph. To aid in the process of data analysis and reduction, Calcomp plotter procedures were developed to automatically plot and scale the data over the range of speed. The variables plotted by the program are amplitude, bearing force, and phase angles.

Analysis of Sample Gas Turbine

As an example of the influence of the support system on the rotor unbalance response, consider the sample case of a small two bearing jet engine which is required to operate between a design speed of 28,000 rpm and an idle speed of 60 percent of design speed. The rotor-bearing system should be designed so that no critical speeds or excessive rotor amplitudes of motion caused by unbalance exist in the operating speed range.

Fig. 2 represents the first two critical speeds for a small gas turbine as a function of bearing stiffness for a range of stiffness varying from 1000 to 1,000,000 lb/in. To calculate the critical speeds, the rotor was divided up into a number of mass stations and the flexible rotor critical speeds were calculated by a standard type Prohl, Myklestad [13] matrix transfer method. The values of the flexible rotor critical speeds are shown as the dotted lines in Fig. 2. To calculate the rigid body critical speeds and to obtain the mass and inertia data required for the computer program, the rotor total weight and its centroid were

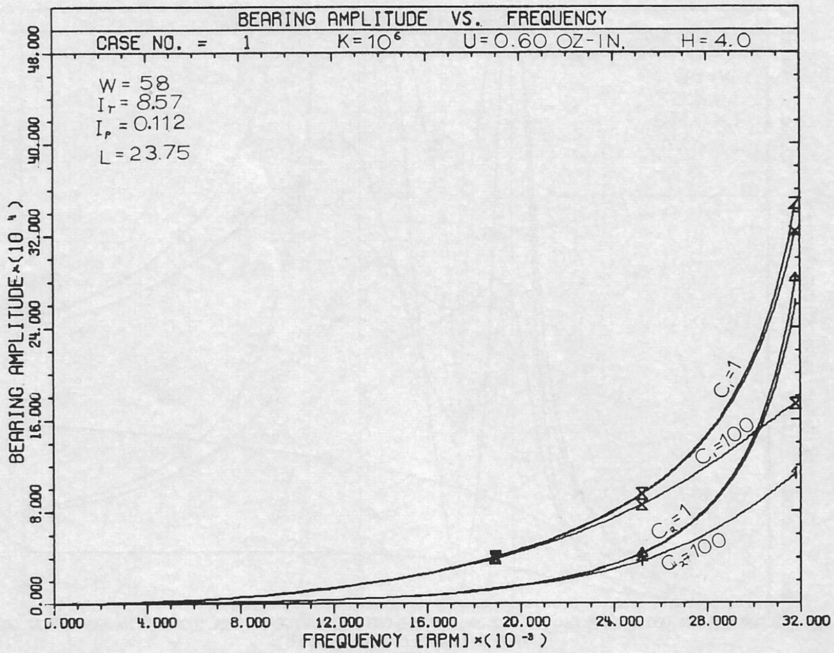

Fig. 3 Bearing amplitude versus frequency for various values of support damping —$K = 10^6$

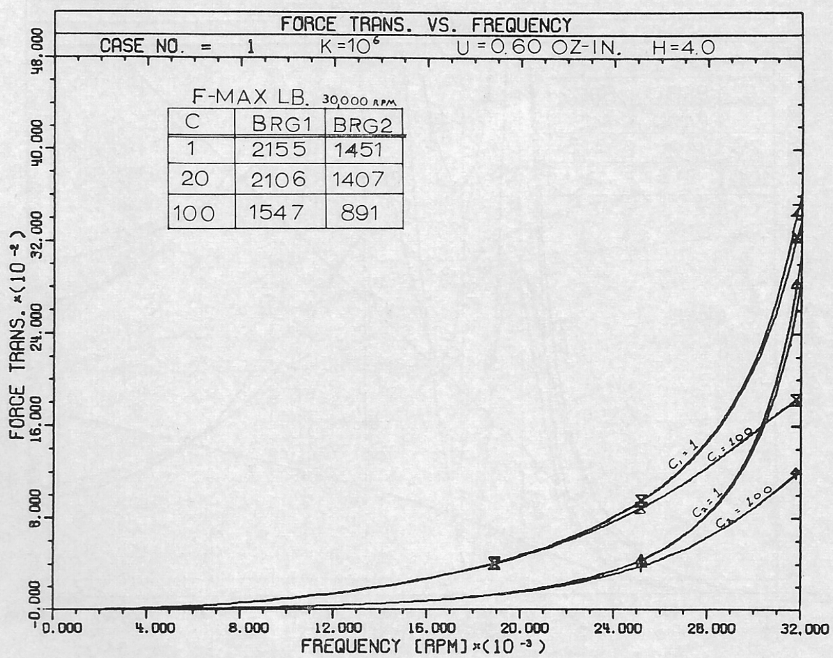

Fig. 4 Force transmitted versus frequency for various values of support damping —$K = 10^6$

determined along with the values of the rotor polar and transverse moments of inertia about the centroid. The solid lines of Fig. 2 represent the rigid body critical speeds for forward synchronous precession. Note that for bearing stiffnesses in excess of 100,000 lb/in., the influence of shaft flexibility causes a reduction in the critical speeds as predicted by rigid body theory. Below 100,000 lb/in., the effect of shaft flexibility should be of minor significance and the rigid body analysis should give accurate results as to rotor amplitude, phase angles, and bearing forces transmitted.

In addition to the requirement of low rotor amplitudes of motion throughout the speed range, the rotor should have long bearing life which is a direct function of the transmitted bearing force. In particular, it is desirable to have an engine which will not suffer a catastrophic failure in the event that a turbine blade is lost. In the following figures, it has been assumed that the turbine blade has failed, resulting in an unbalance of 0.60 oz-in. at an axial distance of four inches from the first bearing.

As a first step in evaluating the rotor performance for various values of support stiffness and damping, the simplest case of isotropic stiffness and damping will be assumed for both bearing supports. In the first case, it will be assumed that the support stiffness is sufficiently high so that the rotor is operating below critical speeds. Fig. 2 shows that if the support stiffness is greater than 1,000,000 lb/in., the rigid body critical speeds are above the design speed range. Fig. 3 represents the amplitude

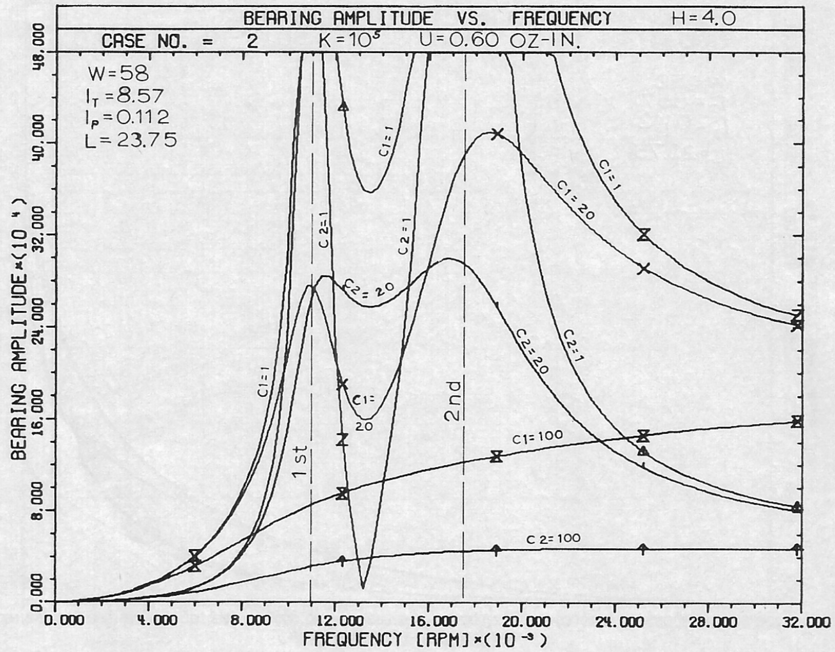

Fig. 5 Bearing amplitude versus frequency for various values of support damping $-K = 10^5$

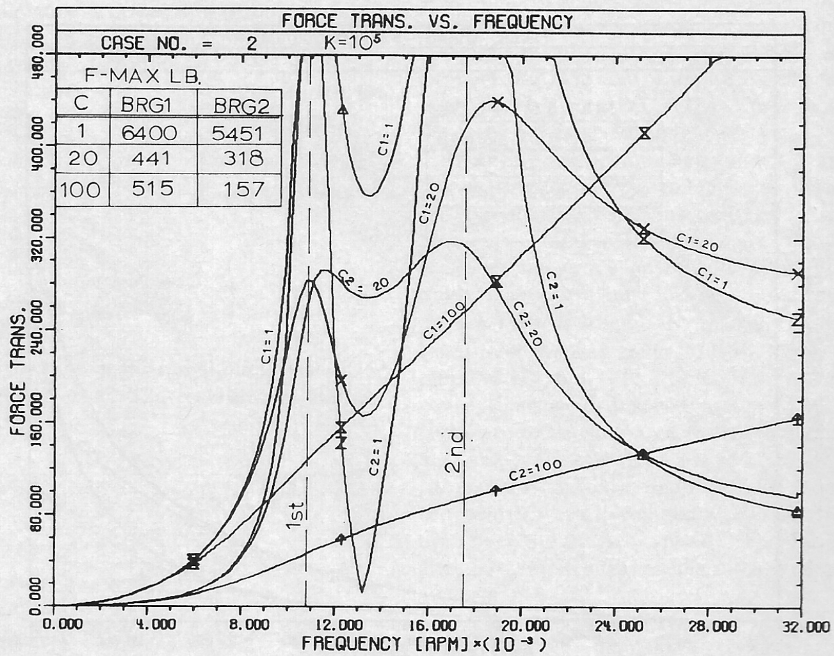

Fig. 6 Bearing force transmitted versus frequency $-K = 10^5$

at the first and second bearings for a support stiffness of $K = 10^6$ lb/in. and for damping values of $C = 1$, 20, and 100 lb-sec/in. The subscript 1 of the damping coefficient C refers to the first bearing while the subscript 2 refers to the second bearing amplitude. Inspection of the amplitude curves shows that, as expected, there are no critical speeds within the operating range. For $C = 1$ the absolute amplitude at the first bearing at 30,000 rpm is 1.9 mils while the second bearing amplitude is 1.3 mils. Increasing the damping from 1 lb-sec/in. to 20 causes little apparent reduction in the rotor amplitude. If the support damping is increased to $C = 100$ lb-sec/in. then the rotor amplitude at the first bearing is reduced to 1.4 mils while the second bearing amplitude is reduced to 0.80 mils.

From the standpoint of rotor amplitude alone, these values of bearing displacements are satisfactory; however, the stiff support system will cause large forces to be transmitted through the bearings. Fig. 4 represents the bearing forces transmitted versus rotor speed for various values of support damping with $K = 10^6$ lb/in. Note that the force transmitted at the first bearing for $C_1 = 1$ is 2155 lb and for $C_1 = 100$ lb-sec/in. this value is reduced only to 1547 lb. Although the rotor amplitude is within acceptable limits in this case, the forces transmitted are excessive at design speed for this high bearing stiffness.

In Case No. 2, Fig. 5, a reduced support stiffness of 100,000 lb/in. was assumed. This places the rotor first and second critical speeds at 11,000 and 17,500 rpm which causes the second critical to fall within the speed operating range. If the support damping is very light, then excessive rotor excursions will occur

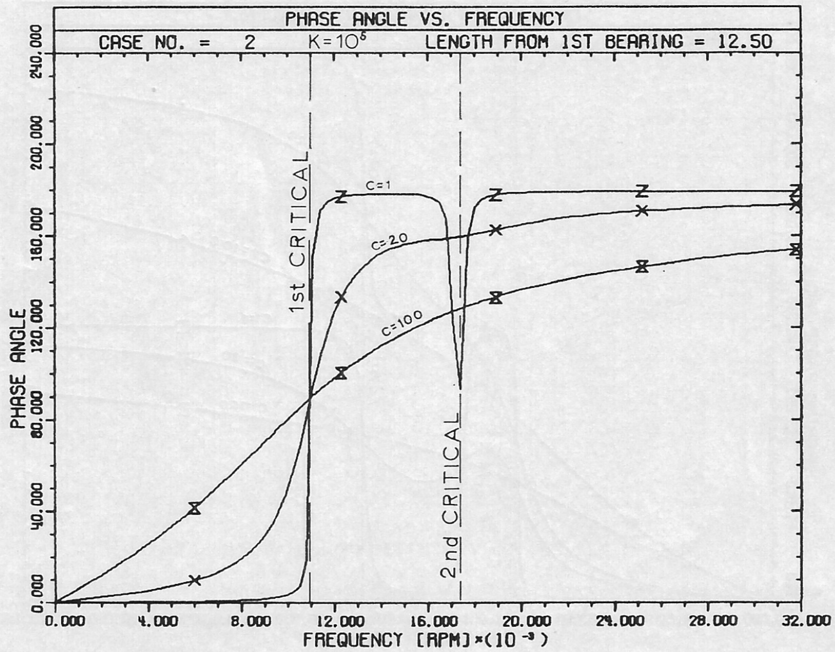

Fig. 7 Phase angle versus frequency at rotor mass center for $K = 10^5$

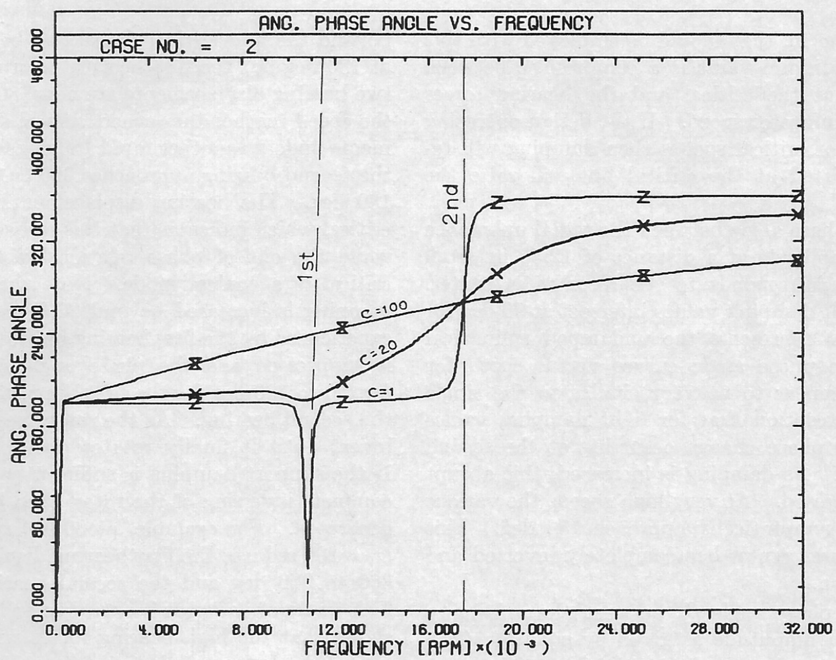

Fig. 8 Angular phase angle versus frequency for $K = 10^5$

at the criticals. For a damping value of $C = 20$, the maximum amplitude is 4.2 mils at the first bearing and occurs at the second critical speed (conical mode) at 18,500 rpm. Note that the support damping has caused a slight increase in the actual critical speed above the value predicted for an undamped system. This is similar to the situation reported in [14] for the single mass model. If the damping is increased still further to 100 lb-sec/in. then there is no observable critical speed response of the system as the amplitudes at both bearings increase smoothly with speed. For example, at 30,000 rpm the motion at the first bearing is 1.6 mils and only 0.5 mils at the second bearing.

Fig. 6 represents a plot of the bearing forces transmitted as a function of rotor speed for various values of damping for $K = 10^5$ lb/in. If the support damping coefficient $C = 1$ then a maximum force of 6400 lbs will be developed at the second critical speed, while a force of over 5400 lb will be transmitted through the second bearing. Thus if the engine were to operate in the speed range of 16 to 20,000 rpm for any significant length of time, bearing damage could be expected. As the damping is increased, the maximum forces developed at the critical speeds decreases. For example with $C = 20$ the maximum force transmitted to the first bearing is reduced to 441 lb. If the bearing damping is increased to 100 lb-sec/in. the force transmitted at 17,500 rpm is reduced to only 260 lb. Fig. 5 shows that with this level of damping in the support system, no critical speed problem will be encountered in the entire speed range. The bearing forces transmitted for $C = 100$ increase with speed and the maximum bearing force of 515 lb at the first bearing occurs at the maximum rotor speed of 30,000 rpm.

Thus it appears that there is an optimum value of damping

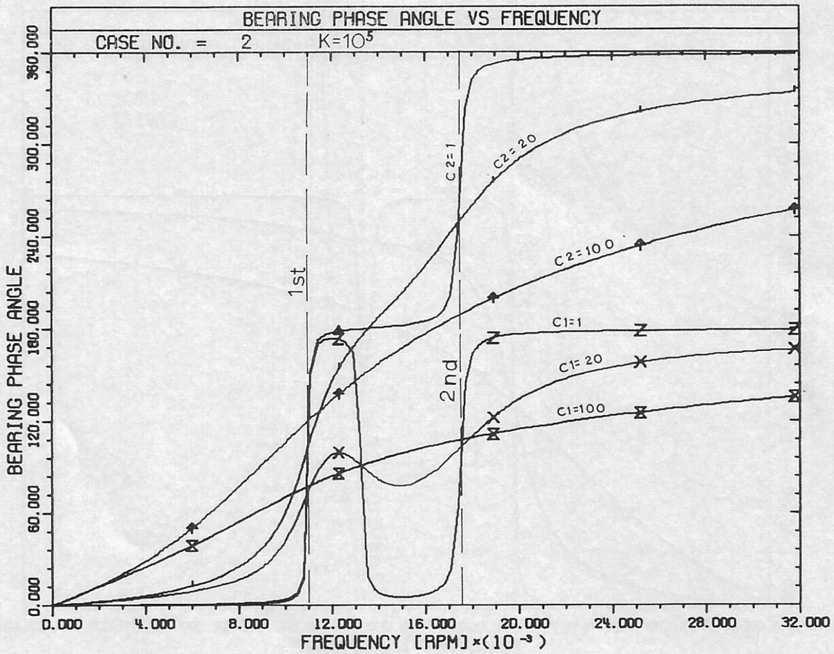

Fig. 9 Bearing phase angle versus frequency —$K = 10^5$

between 20 and 100 lb-sec/in. that should be employed with this stiffness value. This optimum value is a compromise between the forces transmitted at the critical and the bearing forces developed at the maximum rotor speed. If the design operating speed lies above the rotor critical speeds then damping will reduce the forces transmitted at the critical but will raise the values at higher speeds.

Fig. 7 represents the phase angle between the radial unbalance vector and the rotor amplitude at a distance of 12.50 in. from the first bearing which corresponds to the rotor mass center. Note that the lines for all damping values intersect at 90 deg for 11,000 rpm. This point represents the undamped cylindrical natural frequency. The phase angle curves versus speed for the rotor mass center appear to be very similar to the single mass model with the exception that for light damping values there can be an abrupt phase change occurring at the second or conical critical speed. As damping is increased, this abrupt change in phase is suppressed. At very high speeds the various damping lines will all asymptotically approach 180 deg. This implies that the rotor mass center has completely inverted and lies on the axis of rotation.

Fig. 8 represents the phase angle β_α between the unbalance moment and the angular amplitude as given by equation (26). For low values of damping at speeds below the first critical, the angular amplitude and unbalance moment vectors are 180 deg out of phase. At the first or cylindrical critical there is a rapid change in phase as the angle approaches 0 over a very narrow speed range. Note that an increase in support damping will suppress this rapid angular phase angle change at the first critical speed. As the frequency approaches the second critical, which is primarily a conical mode, the curves of various damping all intersect at a point which indicates the angular amplitude is leading the unbalance moment by 90 deg. When the support damping is increased to 100 lb-sec/in., the angular phase angle change proceeds smoothly from 180 to 360 deg as the limiting value.

Fig. 9 represents the displacement phase angles at both bearings with respect to the radial unbalance vector. For light damping where $C = 1$, the motion at both bearings is in phase up to the first critical at 11,000 rpm. At this speed, both bearings undergo a rapid phase angle change of 0 to 180 deg but still remain in phase relative to each other. As the speed is increased beyond the first critical the second bearing phase angle remains at 180 deg but the first bearing returns to 0 degrees. Thus the two bearing displacements are now 180 deg out of phase. When the speed reaches the second critical speed the bearing displacements undergo another rapid 180 deg transition in phase in which the second bearing approaches 360 or 0 deg and the first bearing 180 deg. The bearing displacements are in phase at the first critical which indicates that this is essentially a cylindrical mode while the out of phase relationship indicates that the second critical is a conical mode. Note that as the bearing support damping is increased beyond 20 lb-sec/in. the phase reduction experienced by the first bearing between first and second criticals is suppressed and the phase angle change proceeds smoothly from 0 to 180 deg for the first bearing and from 0 to 360 deg for the second bearing. As the rotor speed increases, the rotor will invert until it finally rotates about its principal inertia axis. If the support damping is sufficiently high so as to prevent the complete inversion of the rotor, then high bearing forces will be generated. For example, at 30,000 rpm the damping value of $C = 100$ reduces the first bearing displacement phase angle from 360 to 260 deg and the second bearing from 180 to 130 deg. This value of damping is excessive and the examination of Fig. 6 shows that the high bearing forces will be transmitted for this case at the design speed.

Rotor Optimum Support Characteristics

In order to determine the optimum bearing support characteristics, a computer program ROTOR 4M was written to evaluate the rotor amplitude and bearing forces transmitted over a given speed range and search for the maximum rotor amplitudes and forces. The program calculates the rotor amplitudes and forces at each speed increment and iterates the speed until the maximum amplitudes or force values at each bearing are obtained. This program was used to evaluate the bearing forces developed for support stiffnesses ranging from $K = 10^3$ to $K = 10^6$ lb/in. and damping values ranging from $C = 1$ to 1000 lb-sec/in.

Fig. 10 represents the maximum bearing force versus damping for various values of support stiffness. The solid line of Fig. 10 represents the force at the rotor critical speed while the dotted line represents the force obtained at the maximum rotor speed of 30,000 rpm. With a high support stiffness value of $K =$

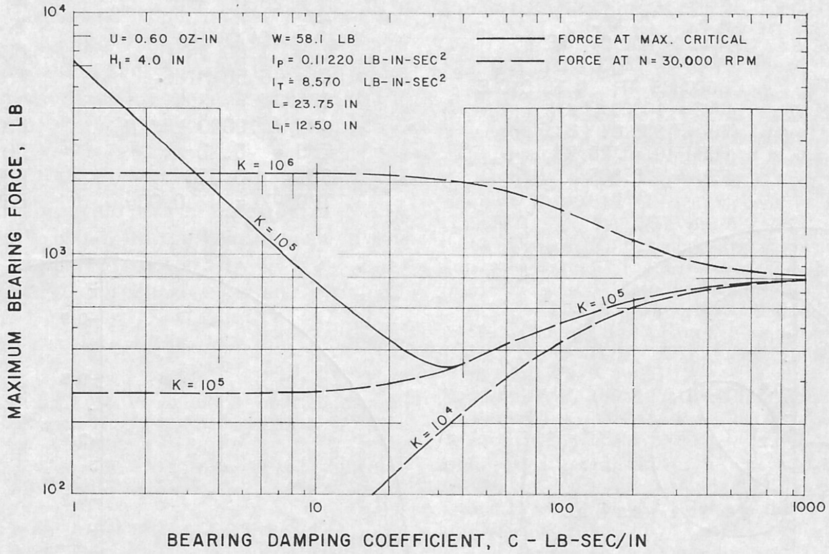

Fig. 10 Maximum bearing force versus damping coefficient for various values of bearing stiffness

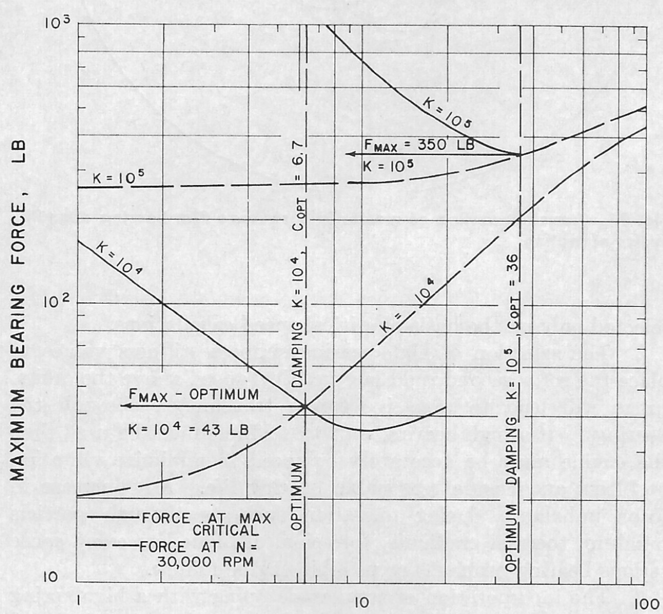

Fig. 11 Maximum bearing force versus damping coefficient for bearing stiffness values $K = 10^4$ and $K = 10^5$

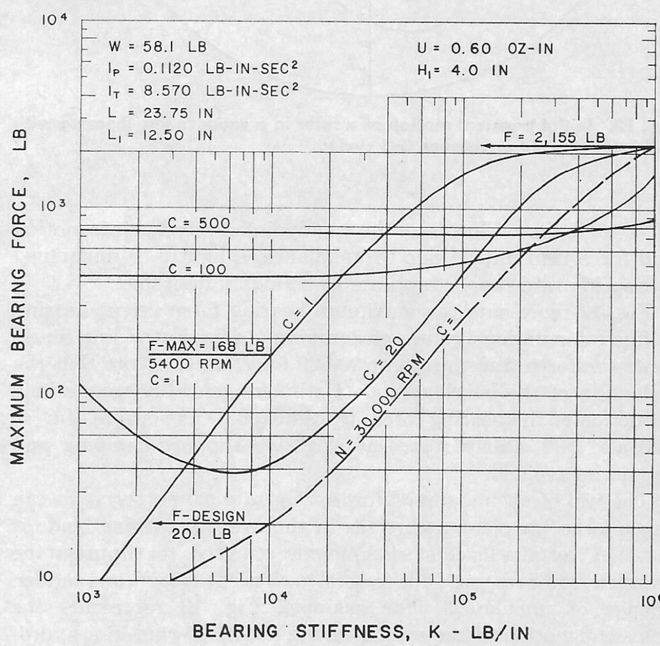

Fig. 12 Maximum bearing force versus bearing stiffness for various values of support damping

10^6 lb/in., the rotor critical speeds will be above the operating speed and the force transmitted will range from 2000 to 800 lb as the damping is increased to 1,000 lb-in./sec. Note that the asymptotic limit of 800 lb as $C > 1{,}000$ represents the force transmitted through a perfectly rigid support. Thus operating below the critical speed with the high support stiffness will generate a greater bearing force than if the bearing and support system were perfectly rigid.

For $K = 10^5$ lb/in., the maximum bearing force occurs at the second critical speed and decreases rapidly as the damping is increased to 30. Above C values of 30 there are no observable critical speeds and the maximum force occurs at the maximum rotor speed. The crossover value at which the force at the critical equals the force at the operating speed is taken as the optimum value.

Fig. 11 is similar to Fig. 10 with the damping range varying from 1 to 100. For the case of $K = 10^5$ lb/in., the force developed at the maximum operating speed will be equal to the force developed at the critical speed for a damping coefficient of $C = 36$ lb-sec/in. causing a value of $F_{\max} = 350$ lb. This damping value may be considered as an optimum and the bearing forces induced by rotor unbalance at any speed will always be less than or equal to $F_{\max}$. If the bearing support stiffness is reduced to 10,000 lb/in. the rotor critical speeds will be reduced to 3500 and 5500 rpm which is well below the idle speed of the engine. The first bearing amplitude will be less than 2 mils for all damping values and the second bearing amplitude will be less than 0.5 mils. For light values of damping, the maximum bearing force will occur at the second critical speed of 5500 rpm. If the rotor speed does not fall below 8000 rpm, the maximum bearing force for $C = 1$ will only be approximately 20 lb as compared to a force transmitted of 2155 lb for the case of $K = 10^6$ lb/in. With an optimum damping value of $C = 6.7$ lb-sec/in. for $K = 10^4$ the maximum force transmitted will not exceed 43 lb over the entire speed range. This may be compared with 800 lb maximum transmitted force for optimum damping with

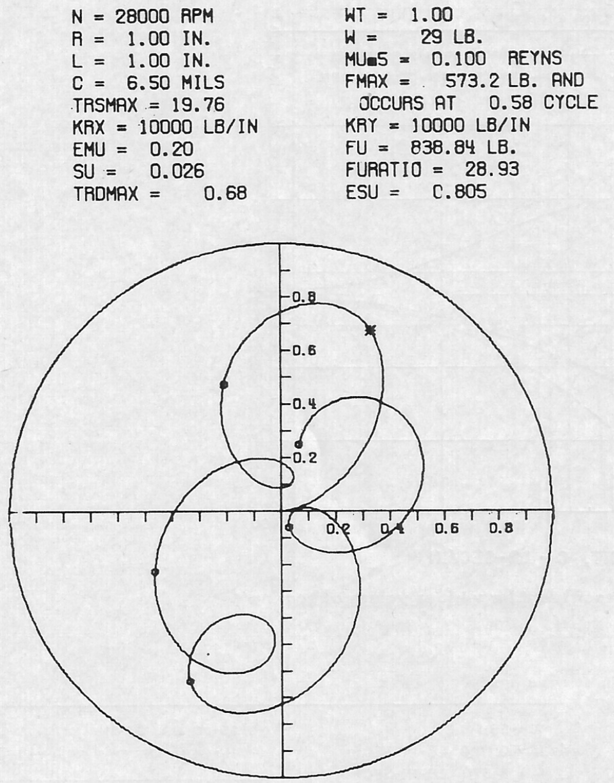

Fig. 13 Initial transient motion of a rotor in a squeeze film bearing with suddenly applied unbalance 0–5 cycles

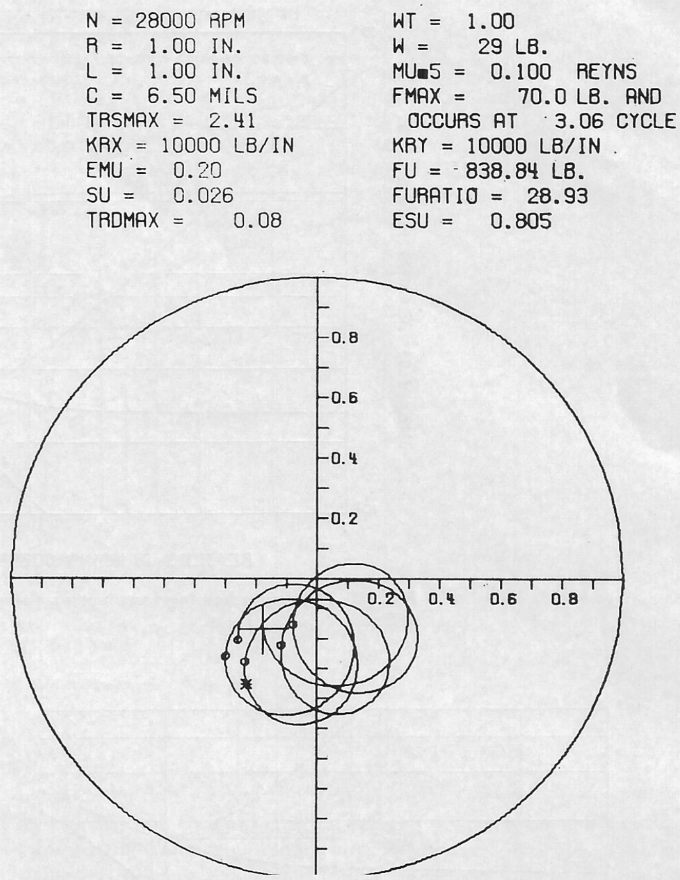

Fig. 14 Transient motion of a rotor in a squeeze film bearing after 20 cycles of motion

$K = 10^6$ lb/in. Thus from the standpoint of steady state motion and force transmitted due to unbalance, a lightly or undamped low spring rate support appears to be highly desirable.

Fig. 12 represents the maximum bearing force versus bearing stiffness for various values of damping. The dotted line represents the force transmitted at 30,000 for $C = 1$. Note that the reduction of the bearing support stiffness generally results in a reduction of the bearing forces transmitted. The curves of $C =$ 100 and 500 clearly represent excessive support damping and should be avoided.

The use of an undamped low spring rate support system can cause large transient excursions in the event a turbine blade is lost. A certain level of damping is required to suppress the transient rotor response due to acceleration, shock load, or a sudden change of unbalance. For example, Fig. 13 represents the transient motion of the rotor running at 28,000 rpm in a hydrodynamic squeeze film bearing with a bearing mounting spring rate of 10,000 lb/in. The rotor is assumed to be initially balanced and upon loss of the turbine blade a large rotor transient motion is experienced causing a maximum force of 573 lb or a dynamic transmissibility of TRD $= 0.68$ after $^1/_2$ shaft cycle of motion. The maximum transient displacement is 80 percent of the squeeze film bearing clearance or over 5 mils excursion. After 20 cycles of shaft motion the maximum force is reduced to 70 lb or a maximum dynamic transmissibility coefficient of only 0.08. From Fig. 11 for $F_{\max} = 70$ lb and $K = 10^4$ the squeeze film bearing is behaving as an equivalent support damping coefficient of $C = 11$ lb-sec/in. after the transient motion has subsided. The basic behavior of the squeeze film bearing is nonlinear and analysis has indicated that for certain levels of speed or unbalance, the squeeze film bearing can actually cause greater forces to be transmitted through the bearing than would a rigid support. The characteristics of the nonlinear squeeze film bearing will be examined in greater detail in Part II.

Summary and Conclusions

1 The bearing support stiffness characteristics should not be selected only on the basis of critical speed calculations.

2 The selection of high bearing support stiffness values to place the rotor second rigid body critical speed above the design range will lead to excessive forces transmitted through the bearings. If a high bearing support stiffness must be used then the engine must be accurately balanced to minimize vibration problems and ensure reasonable bearing life. Any increase in rotor unbalance during operation such as through particle build-up, thermal gradients, corrosion, or blade loss could cause serious bearing problems or possible engine failure.

3 The incorporation of support damping with a high spring rate support system is relatively ineffective in reducing the bearing forces transmitted. Lower support stiffnesses, with moderate damping is quite effective in minimizing bearing forces. For a given support spring rate there is an optimum value of support damping to use. Excessive amounts of damping cause the rotor forces to increase rapidly with speed.

4 The rotor may operate at a critical speed if sufficient damping is incorporated into the support system.

5 A value of support stiffness may be selected to drop the first two critical speeds below the idle speed so that no critical speed problem is encountered throughout the entire speed range.

6 From the standpoint of synchronous unbalance response only, an undamped low stiffness rate support may be used to minimize the rotor forces transmitted over the entire speed range.

7 The use of low stiffness undamped springs for rolling element rotors may lead to stability problems as reported by Alford [1]. Cross coupling forces due to rotor internal friction and the rotor power level may cause large nonsynchronous whirl motion and even possible rotor failure. Thus there exists a lower limit to the values of stiffness and damping that should be used to insure adequate rotor stability.

8 The linear analysis as applied to a squeeze film bearing is only valid for small displacements. The squeeze film bearing support which is used on a number of gas turbine designs is highly nonlinear in its behavior and will not function under certain ranges of unbalance and unidirectional loading.

Acknowledgments

The author wishes to acknowledge the support and encouragement of Robert L. Cummings and Erwin Zaretsky of NASA Lewis Research Center, Cleveland, on the development of this paper. The study was supported in part by NASA Grant NGR 47-005-050 and NASA Institutional Grant NsG-682, University of Virginia.

References

1 Alford, J. S., "Protecting Turbomachinery From Self-Excited Rotor Whirl," *Journal of Engineering for Power*, Trans. ASME, Series A, No. 4, Oct. 1965, pp. 333–344.

2 Kulina, M., "A New Concept for Critical Speed Control," SAE National Aeronautic Meeting, New York, N. Y., April 24–27, 1967.

3 Dworski, J., "High-Speed Rotor Suspension Formed by Fully Floating Hydrodynamic Radial and Thrust Bearings," *Journal of Engineering for Power*, Trans. ASME, Series A, No. 2, Apr. 1964, pp. 149–160.

4 "New Squeeze Bearing Mount," Rocketdyne Corporation, Product Engineering, March 28, 1960.

5 Van Nimwegen, Robert R., "Critical Speed Problems Encountered in the Design of High Speed Turbomachinery," SAE, Oct. 1964, pp. 524–536.

6 Hamburg, G., and J. Parkinson, "Gas Turbine Shaft Dynamics," *SAE Transactions*, Vol. 70, 1962, pp. 774–784.

7 Suter, P., "Bearing Flexibility and Damping at the First Critical Speed," *Sulzer Technical Review 3*, 1962.

8 Linn, F. C., and Prohl, M. A., "The Effect of Flexibility of Support Upon the Critical Speeds of High-Speed Rotors," *Trans. SNAME*, Vol. 59, 1951, pp. 536–553.

9 Cooper, S., "Preliminary Investigation of Oil Films for Control of Vibration," *Proceedings of the Lubrication and Wear Convention*, I. Mech. E. 1963, London, England.

10 *Aviation Week* and *Space Technology*, Feb. 6, 1967, p. 39.

11 Wood, W. J., "Non-Linear Vibration Damping Functions for Fluid Film Bearings," SAE Paper, 1966.

12 Ng, C. W., and Orcutt, F. K., "Steady State and Dynamic Properties of the Floating-Ring Journal Bearing," Journal of Lubrication Technology, Trans. ASME, Series F, Vol. 90, No. 1, Jan. 1968, pp. 243–253.

13 Myklestad, N. O., "A New Method of Calculating Natural Modes of Uncoupled Bending Vibration of Airplane Wings and Other Types of Beams," *Journal of Aeronautical Sciences*, Apr. 1944, pp. 153–162.

14 Gunter, E. J., "Dynamic Stability of Rotor-Bearing Systems," NASA SP-113, Office of Technical Utilization, U. S. Government Printing Office, Washington, D. C., 1966.

Printed in U. S. A.