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Optimum Bearing and Support Damping for Unbalance Response and Stability of Rotating Machinery

This paper presents a rapid approximate method for calculating the optimum bearing or support damping for multimass flexible rotors to minimize unbalance response and to maximize stability in the vicinity of the rotor first critical speed. A multimass rotor is represented by an equivalent single-mass model for purposes of the analysis. The optimum bearing damping is expressed as a function of the bearing stiffness and rotor modal stiffness at the rigid bearing critical speed. Stability limits for aerodynamic cross coupling and viscous internal rotor friction damping are also presented. Comparison of the optimum damping obtained by this approximate method with that obtained by full scale linearized transfer matrix methods for several rotor-bearing configurations shows good agreement. The method has the advantage of being quickly and easily applied and can reduce analysis time by eliminating a time consuming search for the approximate optimum damping using more exact methods.

1 Introduction

A large body of knowledge and sophisticated analytical methods exists today for analyzing the dynamic performance of turbomachinery. Two of the most important problem areas are unbalance response and stability associated with the first bending mode of the rotor. It is, therefore, highly desirable to have an easily applied method to obtain an estimate of the optimum bearing damping for minimum unbalance response and maximum stability when operating above the first critical speed.

Several important advantages accrue from such a method. Machine designers could tentatively select bearing configurations which will give adequate damping without resorting to large numbers of computer aided analyses during the initial design period. Also those engineers called upon to correct abnormal unbalance response or stability problems in machines already constructed and in service, could quickly determine whether modifications to the bearings would present an adequate "quick fix" solution. These capabilities would in many cases reduce the amount of time required to put a machine back into service with a corresponding cost savings to turbomachine manufacturers and users.

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The ability to determine the optimum bearing damping becomes more important as machine operating speeds increase due to lighter, more flexible rotors and more widespread use of tilting pad and other "antiwhirl" bearings. The lower resultant first critical speeds and the greater destabilizing effects of seals, turbine aerodynamics, and internal friction damping necessitate that adequate bearing or support damping be incorporated into the system. The problem is further complicated when preloaded tilting pad bearings are used because the damping remains fairly constant with machine speed while stiffness increases.

An approximate method is presented whereby the optimum bearing or support damping can be calculated as a function of the rotor and bearing stiffness properties. It is particularly applicable to bearings that have minimal cross-coupling effects such as tilting pad and squeeze film bearings.

2 Optimum Bearing Damping

As a first step in deriving an explicit expression for the optimum bearing damping, the multimass flexible rotor is represented as an equivalent single mass rotor for analysis in the vicinity of the first flexible rotor critical speed (Fig. 1). This model constitutes a modal representation, and the modal mass and stiffness of the rotor are obtained from energy considerations [1].[1]

Hence, the modal mass and rotor stiffness are

[1] Numbers in brackets designate References at end of paper.

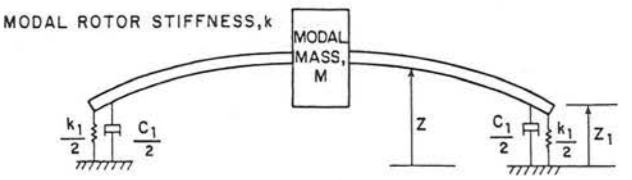

Fig. 1 Modal representation of multimass flexible rotor for analysis near the first critical speed.

$$M = \Sigma m_i \phi_i{}^2 \tag{1}$$

$$k = M\omega_{cr}{}^2 \tag{2}$$

where ω_{cr} is the rigid bearing critical speed.

Then, for the rotor in flexible damped bearings undergoing forced vibration

$$\ddot{Z} + \omega_{cr}{}^2(Z - Z_1) = e_u\omega^2 e^{i\omega t} \tag{3}$$

$$2\xi_1\omega_{cr}\dot{Z}_1 + (K\omega_{cr}{}^2)Z_1 + \omega_{cr}{}^2(Z_1 - Z) = 0 \tag{4}$$

where it has been assumed that the only appreciable rotor damping is obtained from the bearings, the coupling in the horizontal and vertical directions is negligible and that bearings are essentially identical.

By assuming synchronous motion and solving for Z_1 in terms of Z equations (3) and (4) are combined to give

$$\ddot{Z} + 2\omega_{cr}\xi_e\dot{Z} + \Omega^2 Z = e_u\omega^2 e^{i\omega t} \tag{5}$$

where

$$\xi_e = \frac{\xi_1}{(1+K)^2 + (2f\xi_1)^2} \tag{6}$$

$$\Omega^2 = \omega_{cr}{}^2\left[\frac{K(1+K) + (2f\xi_1)^2}{(1+K)^2 + (2f\xi_1)^2}\right] \tag{7}$$

Since many machines operate with $K > 2$, it is assumed that the critical speed on flexible bearings does not differ greatly from that on rigid bearings, i.e., $f = 1$. As the stiffness ratio, K, becomes larger the assumption becomes less of an approximation, and it is for large K values that the need for optimum damping becomes most critical. Hence, the effective rotor damping may be maximized with respect to the bearing damping by finding the value of ξ_1 which satisfies $\partial\xi_e/\partial\xi_1 = 0$. Hence

$$\xi_{em} = \frac{1}{4(1+K)} \tag{8}$$

$$\xi_{10} = \frac{1+K}{2} \tag{9}$$

Using these expressions the approximate rotor amplification factor due to unbalance is given by

$$A = 2(1+K) \tag{10}$$

Equation (5) and the corresponding free, damped vibration equation have been solved exactly to determine the bearing damping which minimizes the unbalance response and which maximizes the real part of the damped eigenvalue. A comparison of the exact solutions and the approximate solution given by equation (9) is shown in Fig. 2. It is seen that the approximate solution provides a very reasonable estimate of the optimum damping for a wide range of the stiffness ratio, K. The expression for the maximum effective damping, equation (8), corresponds to that obtained by Black [2] using a perturbation solution technique.

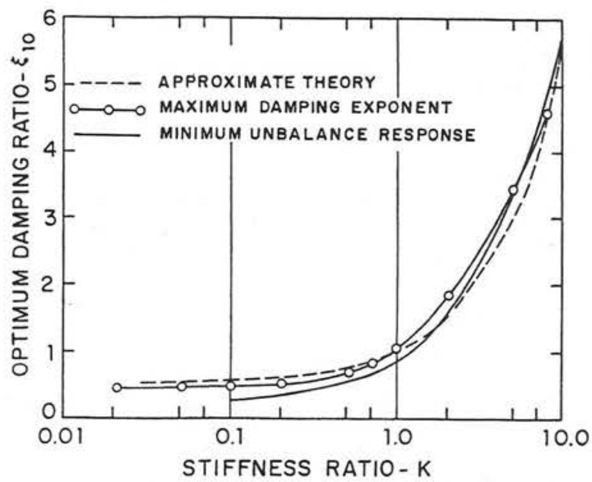

Fig. 2 Comparison of optimum bearing damping obtained from approximate method and exact solution of modal equations of motion

Table 2 shows the ratio of bearing stiffness for the four example rotors considered in this study. These rotors were chosen on the basis of their availability to the authors rather than for the large values of K shown. However, these examples do show that the foregoing analysis is applicable to many rotors.

Fluid film bearing stiffness and damping coefficients are often presented in nondimensional form as functions of the bearing Sommerfield number, the nondimensionalization being

$$\overline{K}_b = \frac{K_b c}{W} \tag{11}$$

$$\overline{C}_b = \frac{C_b \omega c}{W} \tag{12}$$

If the nondimensional stiffness coefficient is known at the first critical speed, the optimum nondimensional damping coefficient is

$$\overline{C}_{bo} = \left(\frac{c\omega_{cr}{}^2}{2g}\right) + \overline{K}_b \tag{13}$$

This value can be quickly compared with the actual nondimensional damping coefficient to determine how effective the bearing will be at the first critical speed.

3 Stability With Optimum Damping

Rotor bearing systems are frequently subjected to self excited instability mechanisms including bearings, seals, aerodynamic effects,

Nomenclature

A = rotor amplification factor (Z/e_u) Dim
c = bearing clearance L
C = damping FTL^{-1}
C_i = internal viscous friction damping FTL^{-1}
e_u = modal mass unbalance eccentricity L
k = fundamental equivalent shaft stiffness FL^{-1}
K = stiffness ratio, k_1/k Dim
m = total rotor mass FT^2L^{-1}
M = rotor modal mass FT^2L^{-1}
q = aerodynamic cross coupling FL^{-1}
$Q = q/m\ T^{-2}$
W = rotor modal weight F
ξ = damping ratio, $C/2m\ \omega_{cr}$ Dim
Z = displacement L
ϕ = normalized rotor mode shape Dim
ω = rotor speed T^{-1}
ω_{cr} = rotor rigid bearing critical speed T^{-1}

Subscripts

1 = bearing property
2 = support property
b = bearing
e = effective property
m = maximum value
o = optimum value
s = support

Table 1 Summary of modal data for example industrial rotors

Rotor	Modal Weight (N)	Rigid Bearing Critical Speed (RPM)	Operating Speed (RPM)	Modal Shaft Stiffness (N/cm x 10^{-5})
Light Rotor [10]	2106	5100	Above First Critical	3.50
Heavy Rotor [10]	24761	2540	Above First Critical	10.20
8-stage Compressor [12]	3002	3821	10,000	2.80
SSME-HPFTP [13]	285	16500	28,000	4.96

Table 2 Summary of optimum damping data for example industrial rotors

Rotor	Bearing Stiffness (N/cm x 10^{-6})	Bearing Damping (N-s/cm)	Stiffness Ratio, K	Optimum Brg. Damping (N-s/cm)	Support Stiffness (N/cm x 10^{-6})	Opt. Support Damping (N-s/cm)
Light Rotor [10]	1.04	2942	3.4	2531	Rigid Support	---
	1.82	4432	5.9	3982	Rigid Support	---
Heavy Rotor [10]	25.98	Not Given	29.2	10107	Rigid Support	---
8-stage Compressor [12]	2.50	2559	10.2	6862	Rigid Support	---
	2.50	2559	0.36	----	0.088	528
SSME-HPFTP [13]	Rigid Bearings	----	0.71	----	0.306	425
	Rigid Bearings	----	2.02	----	0.876	776
	Rigid Bearings	----	4.04	----	1.751	1264

and internal rotor friction damping [3–9]. The analysis just presented considered the bearings to be isotropic and to have negligible cross coupling. Thus bearings are not considered to be a source of rotor instability. The effects of the optimum bearing damping obtained from such a system on the stability limits with aerodynamic cross coupling or viscous internal friction will now be considered.

For free, damped vibrations with aerodynamic cross coupling and optimum bearing damping, equation (5) becomes

$$\ddot{Z} + 2\omega_{cr}\xi_{em}\dot{Z} + (\Omega^2 - iQ)Z = 0 \tag{14}$$

where ξ_{em} is given by equation (8) and

$$\Omega^2 = \frac{\omega_{cr}{}^2(1 + 2K)}{2(1 + K)} \tag{15}$$

With viscous internal friction damping, equation (5) becomes

$$\ddot{Z} + 2\omega_{cr}(\xi_{em} + \xi_i)\dot{Z} + (\Omega^2 - 2i\omega_{cr}\omega\xi_i)Z = 0 \tag{16}$$

where ξ_i is the internal friction damping ratio.

The solution of the eigenvalues of equations (14) and (16) at the instability threshold show the whirl frequency to be Ω. The maximum permissible aerodynamic cross coupling with optimum bearing damping is

$$Q_{\max} = \frac{\omega_{cr}{}^2}{2(1 + K)}\left[\frac{1 + 2K}{2(1 + K)}\right]^{1/2} \tag{17}$$

whereas with viscous internal friction the rotor will only be stable if the operating speed is

$$\omega < \frac{\omega_{cr}}{(1 + K)^2}\left[\frac{1 + 2K}{2}\right]^{1/2}\left[1 + \frac{K + 1}{4\xi i}\right] \tag{18}$$

4 Application to Turbomachinery

As a first example of the application of the preceding analysis, consider the ten-stage centrifugal compressor described in reference [10] and designated therein as the "light" rotor. The unit is nearly symmetrical and weighs 4212 N (947 lb). The rotor is supported in two five-pad tilting pad bearings with the following dimensions: L = 38.9 mm (1.53 in.), D = 101.6 mm (4 in.), and C_d = 0.1524 mm (0.006 in.). The load is acting on the pad.

Assuming C_d to be the bearing diametral clearance and the preload to be 0.5, the radial pad clearance is C_P = 0.1016 mm (0.004 in.). If we further assume the pad arc length to be 60 deg, the offset factor to be 0.5 and the viscosity to be 9.0×10^{-3} Pa − s (1.3×10^{-6} lb-s/in.^{2}), the nondimensional stiffness and damping coefficients at the rigid bearing critical speed of 5100 rpm are

$\overline{K}_{xx} = 5.04$
$\overline{K}_{yy} = 8.78$
$\overline{C}_{xx} = 7.58$
$\overline{C}_{yy} = 11.42$

Assuming the modal mass to be one half the total rotor mass, the stiffness coefficients are $k_{xx} = 1.04 \times 10^6$ N/cm (5.96×10^5 lb/in.) and $k_{yy} = 1.82 \times 10^6$ N/cm (1.04×10^6 lb/in.) which are similar to those used in reference [10]. Applying equation (13) the optimum nondimensional damping coefficients are

$\overline{C}_{xxo} = 6.52$
$\overline{C}_{yyo} = 10.26$

Thus the bearings produce damping values within 16 percent of the optimum. The optimum damping values could have been found by use of equation (9) directly by calculating the dimensional bearing stiffness values and the effective rotor stiffness based on the modal mass and rigid bearing critical speed. The bearing to shaft stiffness ratios are K = 3.4 in the horizontal direction and K = 5.9 in the vertical direction. The rotor will be stable from aerodynamic effects if the total effective aerodynamic cross coupling does not exceed 35000 N/cm (20000 lb/in.).

As a second example, consider the seven-stage centrifugal compressor designated in reference [10] as the "heavy" rotor. This is a 49341 N (11093 lb) unit mounted in pressure dam journal bearings. The rigid bearing critical speed is approximately 2540 rpm and the modal weight is approximately 24761 N (5546 lb). Hence, the effective shaft stiffness is approximately $k = 1.779 \times 10^6$ N/cm (1.016×10^6 lb/in.). The average vertical bearing stiffness is given in reference [10] as $k_{yy} = 2.598 \times 10^7$ N/cm (1.484×10^7 lb/in.). Hence, the stiffness ratio is K = 29.2 and the optimum bearing damping is C_{bo} = 10107 N-s/cm (57715 lb-s/in.).

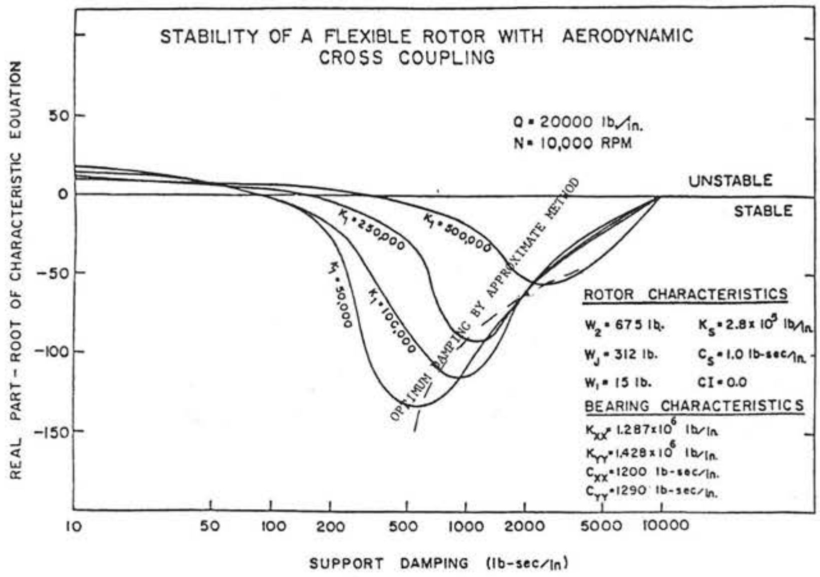

Fig. 3 Comparison of optimum support damping for eight-stage centrifugal compressor obtained by approximate method and exact solution of the multimass equations of motion

The damping values for these bearings was not given in the reference and insufficient data was given to calculate them, however even if the optimum damping were present the approximate rotor amplication factor given by equation (10) is $A = 60$. Thus the rotor is extremely sensitive to any unbalance that will excite the first mode and it would also require only 1926 N/cm (1100 lb/in.) effective aerodynamic cross coupling to render the rotor unstable. With the extremely stiff bearings the flexible bearing critical speed is very close to the rigid bearing critical speed and even optimum bearing damping has little effect since the bearing amplitudes are very small.

The third example considers an eight-stage centrifugal compressor mounted in tilting pad bearings. The rotor weighs 6005 N (1350 lb) and has a rigid bearing critical speed of 3821 rpm. The effective rotor stiffness is 4.903×10^5 N/cm (2.8×10^5 lb/in.). In the vertical direction the bearing stiffness and damping coefficients are $K_{yy} = 2.5 \times 10^6$ N/cm (1.428×10^6 lb/in.) and $C_{yy} = 2259$ N-s/cm (1290 lb-s/in.). The vertical stiffness ratio is, therefore, $K = 10.2$. The optimum bearing damping is $C_{yyo} = 6862$ N-s/cm (3920 lb-s/in.) which is over three times the damping actually provided by the bearings.

Based upon a linearized stability analysis and actual testing, the rotor was found to have an effective aerodynamic cross coupling of $q = 35000$ N/cm (20000 lb/in.). Even with optimum damping, equation (17) shows that the rotor will be unstable if the cross coupling exceeds 21364 N/cm (12200 lb/in.).

In order to stabilize the rotor, a parametric analysis using a flexible damped support (squeeze film damper bearing) in series with the tilting pad bearings was performed. The results are illustrated in Fig. 3 where the real part of the system first forward mode eigenvalue is plotted versus support damping for several values of support stiffness. To obtain these curves, the rotor was modeled as a six-degree-of freedom system for analysis in the vicinity of the first mode and the dynamical equations of motion were solved for the twelfth order characteristic polynomial [11–12]. This polynomial was then numerically solved for the system eigenvalues for each combination of the bearing and support properties considered. It required approximately 12 computer runs to describe each curve in Fig. 3 to obtain the optimum support damping for each support stiffness considered. Thus the parametric stability analysis required a substantial amount of time to prepare. It is of interest to compare the values of optimum damping obtained from the more complex parametric analysis with those found using the method in this paper.

Because the squeeze film damper bearings are in series with the tilting pad bearings, the combined effective stiffness and damping of squeeze film-tilting pad combination must be used in the optimum damping calculations. The equations used for this are derived in the Appendix. With a squeeze film support stiffness of 87559 N/cm (50000 lb/in.) the effective bearing-support stiffness is 84894 N/cm (48478 lb/in.) and the optimum total effective bearing-support damping is 824 N-s/cm (471 lb-s/in.). Using the equations in the Appendix, the squeeze film support must have a damping of 925 N-s/cm (528 lb-s/in.). This corresponds very closely to the optimum support damping value of 936 N-s/cm (550 lb-s/in.) obtained from the parametric stability study. Repeating the process for other values of support stiffness results in the dashed curve in Fig. 3 which shows very good agreement with the parametric study over the wide range of support stiffness considered and was obtained much more quickly. With the optimum support damping and support stiffness of 8.76×10^4 N/cm (5×10^4 lb/in.) the maximum aerodynamic cross coupling has been increased from 21364 to 144385 N/cm (12200–82450 lb/in.).

The final example is the Space Shuttle Main Engine—High Pressure Fuel Turbopump (SSME–HPFTP). The HPFTP consists of a three-stage centrifugal pump section and a two-stage turbine section as shown in Fig. 4. Details of the pump configuration are given in reference [13].

The rotating assembly is supported in flexibly mounted ball bearings and is acted on by aerodynamic cross-coupling forces in the turbine section. In its original configuration, little damping was provided to the rotor from the bearings, supports, and seals.

Fig. 5 illustrates the results of a stability analysis performed on the HPFTP considering the turbine aerodynamics to be the prime exci-

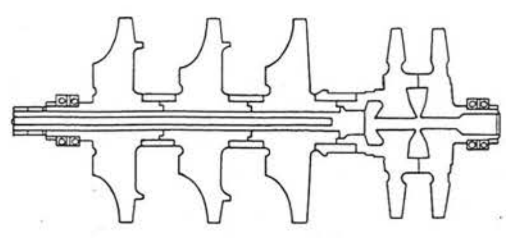

Fig. 4 Space Shuttle Main Engine—High Pressure Fuel Turbopump (SSME—HPFTP)

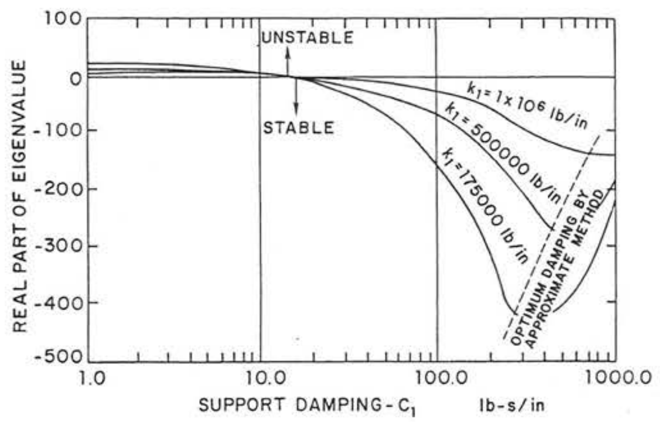

Fig. 5 Comparison of optimum ball bearing support damping for SSME-HPFTP obtained by approximate method and transfer matrix method with q = 2.63 × 10⁴ N/cm (15000 lb/in.)

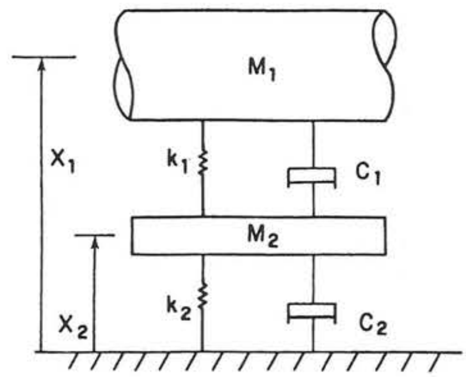

Fig. 6 Bearing mounted on flexible, damped support

tation. The supports were considered symmetric and isotropic. The operating speed is 28000 rpm. Fig. 5 shows the results of varying the total effective bearing-support damping for values of support stiffness from 3.065×10^5 to 1.751×10^6 N/cm (1.75×10^5–1.0×10^6 lb/in.). This analysis was performed by a linearized stability technique using a transfer matrix approach [14]. As in the previous example, a large amount of time was spent running the linearized stability program to obtain the optimum damping values. Approximately 30 computer runs were required to produce the map in Fig. 5.

To use the method described in this paper, the rigid bearing critical speed was calculated to be 16500 rpm and the modal mass was found to be 284.7 N (64.0 lb). Hence the effective rotor stiffness is 8.67×10^5 N/cm (4.95×10^5 lb/in.). The bearing/support stiffness ratios are K = 0.71, 2.02, and 4.04 and the corresponding optimum effective support damping values are 425, 776, and 1264 N-s/cm, respectively, (243, 433, and 722 lb-s/in.). These values are shown by the dashed curve in Fig. 5 and agree closely with the optimum values predicted by the linear stability analysis. The maximum permissible aerodynamic cross coupling values at each support stiffness value with optimum damping are 2.13×10^5, 1.31×10^5 and 8.16×10^4 N/cm (1.22×10^5, 7.49×10^4 and 4.66×10^4 lb/in.). These values are in excess of the value of 2.63×10^4 N/cm (15000 lb/in.) considered in the stability analysis. A summary of these examples is given in Tables 1 and 2.

5 Conclusions

An expression for the optimum bearing damping has been developed as a function of the bearing/rotor stiffness ratio. The bearings were assumed to have negligible cross coupling effects as in tilting-pad and squeeze film damper bearings. The optimum bearing damping is easily calculated knowing only the rigid bearing critical speed and modal weight of the rotor. This information is obtained from undamped critical speed calculations. The optimum damping calculation is valid for both unbalance response at the first rotor critical speed and first mode stability as shown by comparison with the exact solution of the equations of motion. Stability limits with aerodynamic cross coupling and viscous internal frictions damping with optimum bearing damping are expressible as functions of the bearing/rotor stiffness ratio. If the bearings are mounted on flexible damped supports auxiliary equations must be used to determine the support damping necessary to give the optimum total effective bearing-support damping.

The optimum nondimensional bearing damping obtained from hydrodynamic bearing analysis is related to the nondimensional bearing stiffness at the critical speed only by the bearing clearance and rotor rigid bearing critical speed. This makes it very easy to determine whether the bearing design is desirable merely by using tabular hydrodynamic bearing data.

Comparisons of the optimum damping values calculated by the method presented in this paper to those found by the transfer matrix method for a number of rotor systems shows good correlation. Thus, the method is an effective tool to provide bearing-support design information quickly. Application of the method will result in time and cost savings in many instances by eliminating time consuming analysis with nonoptimum bearing designs.

APPENDIX

To calculate the optimum support damping for a rotor with a damped flexible support in series with a bearing, it is necessary to know the total effective support-bearing stiffness and damping. Fig. 6 shows a schematic of the bearing support structure.

The equations of motion are

$$M_1\ddot{x}_1 + C_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0 \qquad (A.1)$$

$$M_2\ddot{x}_2 + C_1(\dot{x}_2 - \dot{x}_1) + C_2\dot{x}_2 + k_1(x_2 - x_1) + k_2x_2 + 0 \qquad (A.2)$$

Assuming solutions of the form $x = Xe^{i\omega t}$ equation (A.2) is solved for x_2 in terms of x_1 and substituted into equation (A.1) giving

$$M_1\ddot{\mathrm{x}}_1 + [C_1(1 - B) - k_1D/\omega]\dot{\mathrm{x}}_1 + [k_1(1 - B) + \omega C_1D]\mathrm{x}_1 = 0 \qquad (A.3)$$

or

$$M_1\ddot{\mathrm{x}}_1 + C_e\dot{\mathrm{x}}_1 + k_e\mathrm{x}_1 = 0 \qquad (A.4)$$

which represent the total effective stiffness and damping of the bearing-support combination. B and D are given by

$$B = \frac{k_1(k_1 + k_2 - m_2\omega^2) + \omega^2C_1(C_1 + C_2)}{(k_1 + k_2 - m_2\omega^2)^2 + \omega^2(C_1 + C_2)^2} \qquad (A.5)$$

$$D = \omega\left[\frac{C_1(k_1 + k_2 - m_2\omega^2) - k_1(C_1 + C_2)}{(k_1 + k_2 - m_2\omega^2)^2 + \omega^2(C_1 + C_2)^2}\right] \qquad (A.6)$$

To determine the optimum damping of the bearing-support combination, k_e is calculated neglecting the support damping C_2. This value is used to determine the stiffness ratio, K, and hence, the total optimum bearing-support damping. Since C_1, k_1, and k_2 are constant, the value of C_2 giving the optimum bearing-support damping is given by

$$C_2 = \frac{-B_1}{2A_1} \pm \left[\left(\frac{B_1}{A_1}\right)^2 - \frac{D_1}{A_1}\right]^{1/2} \qquad (A.7)$$

where

$$A_1 = \omega^2(C_1 - C_{eo}) \qquad (A.8)$$

$$B_1 = \omega^2C_1(C_1 - 2C_{eo}) + k_1^2 \qquad (A.9)$$

$$D_1 = (k_1 + k_2 - m_2\omega^2)^2 - k_1C_1(k_1 + 2k_2 - 2m_2\omega^2) - \omega^2C_1^3 \qquad (A.10)$$

The plus sign in equation (A.7) is used when B_1/A_1 is positive. The effect of C_2 on the total effective stiffness should then be checked to see if the effective damping is near the optimum. In some cases several iterations are necessary to calculate the support damping that optimizes the bearing-support structure since the support damping affects the effective stiffness.

The application of these equations takes into account the lost effectiveness of the support damping when transmitted through the bearings. If the bearing stiffness is much greater than the support stiffness, the effective total bearing-support stiffness neglecting support damping can be closely approximated by the support stiffness. However, the support damping required to give the optimum total effective bearing-support damping must still be calculated using equation (A.7) to retain sufficient accuracy.

In some cases with large support stiffness, k_2, the optimum effective bearing-support damping cannot be achieved by increasing the support damping. This occurs because the support damping increases the total effective bearing-support stiffness which decreases the damping transmitted through the support and bearing. There is still a value of support damping, C_2, that maximizes the total effective damping transmitted to the rotor, ξ_e. In such cases, calculation of C_e in equation (A.4) for different values of C_2 will indicate the new optimum value.

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